

# Enhanced Techniques For Determining Initial Feasible Solutions In Classical Transportation Models

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## Abstract

*This research paper has presented the effective algorithm for the solution to the Transportation Problem (TP), with an emphasis the importance of a properly formulated constraint system. The main purpose of this research paper is to introduce a logical and applicative method for obtaining the Initial Basic Feasible Solution (IBFS) to any TP. To demonstrate the effectiveness of the method introduced, a numerical example is presented here. Additionally, a comparison study is also presented to assess the performance of the new method in comparison to traditional methods. The results show that the proposed method gives the better and optimal solution for the mentioned numerical problem.*

**Keywords:** Transportation Problem (TP), Basic Feasible Solution, Excel solver, Transportation Methods, Balanced & Unbalanced Problem.

## 1. Introduction

The Transportation Problem (TP) is a model of Linear Programming Problem (LPP) that focuses on the optimal transportation of a uniform product from the sources like production or supply centers to intended destinations like warehouses or distributors [3]. Each source is limited by a predetermined supply capacity, and each destination is characterized by a predetermined demand requirement. The overall main objective of TP is to find the most efficient allocation of shipment quantities from sources to destinations such that the total transportation cost is minimized, under the constraints of supply and demand [7]. TP generally consists of optimizing the distribution model by making cost-effective transportation to meet all the logistical and operational constraints [13].

The Transportation Problem (TP) is a main group of Linear Programming Problems (LPP) that have a key position in the field of operations research. It mainly focuses with the effective allocation of a single, uniform products from a number of supply points that known as sources, production facilities, or warehouses, to a number of demand points that known as destinations, customers, or distribution centers [5]. Both the supply capacity of each source and the demand requirement of each destination are fixed. The aim of the transportation problem is to

find the economical distribution of products from various sources to multiple destinations so that the overall cost of transportation is optimized, and the supply and demand constraints are balance [11]. Such a problem is highly relevant in real-world logistics and supply chain networks, where source need to transport goods from factories or central depots to distributors, regional distribution centers, or customers at the minimum cost [8]. The normal form of a TP is generally given in a matrix format, with supply points represented by rows and demand points by columns. The unit transportation cost from a particular source to a particular destination is contained in each cell of the matrix. The problem is to complete this matrix with shipment quantities so that the total shipments from each row are equal to the supply at that source, and the total shipments in each column are equal to the demand at that destination, minimizing the total cost.

This research paper is concerned with the first step of solving the transportation problem obtaining the IBFS. This step is particularly important, because an initial solution would be minimized computational costs in the following steps. The method introduced in this work introduces a new and useful methodology for finding the IBFS. Additionally, comparative analysis with current methods is carried out to measure the efficiency, precision, and computational ease of the model proposed. The

result of the mentioned numerical problem that the new method provides an improved faster and efficient approach to optimizing transportation problems, especially in cases where the dataset is large or complicated logistical structures are involved [1][9]. Some of these include the “North-West Corner Rule, Least Cost Method, Vogel's Approximation Method, and the Modified Distribution Method (MODI)”.

## 2. Design & Methodology

TP is a type of LPP that may be solved efficiently through the various transportation method, that is the version of the simplest algorithm [5]. Because it may be used to optimize distribution from multiple sources to multiple destinations, below mentioned method is commonly utilized in operations research. The following step of transportation provides the process easier by using a tabular methodology that improves computational effectiveness and readability by resolving large-scale problems with multiple supply and multiple destination demand points.

### 2.1 Transportation Tabular Form

Transportation problems are structured mathematically in the form of a transportation tableau, a formal matrix that tabulates all parameters of the problem. The tableau is a visualization and analysis tool that concludes the supply capacities, needs of demand, unit transportation prices for each source-destination group. The supply values usually

represented as for every source are shown on the right side of every row, but the demand quantities represented as for each destination are shown at the bottom of every column. Every cell in the matrix is a possible route between a supply point and a demand point and the unit transportation cost for that path. These cells are the decision variables in the model, which represent the number of products to be transport on every path. The tableau effectively contains the constraints of supply and demand, as well as the mentioned cost structure, that has an approach to get optimal solution. It shows the feasible solution through a its representation, allowing decision-makers to evaluate different allocation strategies.

This tabular framework is not only useful in solving the problem but also as a basis for using solution algorithms like the “North-West Corner Rule, Least Cost Method, Vogel's Approximation Method, and the MODI method for optimization”. It is also especially useful to minor changes during iterative solution steps and analyzing changes in supply, demand, or cost parameters. the transportation tableau is a important resource in solving and modeling transportation problems, providing a simple and effective framework to solve distribution logistics in an efficient manner. In presenting the essential supply, demand, and cost elements in a matrix, it facilitates both the generation of the initial feasible solution as well as the optimization phase that follows.

Table 1- Transportation Table

Destination →	D <sub>1</sub>	D <sub>2</sub>	... D <sub>j</sub> ...	D	Disponibilité
↓ source ↓					
S <sub>1</sub>	$c_{11}$ $x_{11}$	$c_{12}$ $x_{12}$		$c_{1m}$ $x_{1m}$	a <sub>1</sub>
S <sub>2</sub>	$c_{21}$ $x_{21}$	$c_{22}$ $x_{22}$		$c_{2m}$ $x_{2m}$	a <sub>2</sub>
... S <sub>i</sub> ...			$c_{ij}$ $x_{ij}$		... a <sub>i</sub> ...
S <sub>n</sub>	$c_{n1}$ $x_{n1}$	$c_{n2}$ $x_{n2}$		$c_{nm}$ $x_{nm}$	a <sub>n</sub>
Demande	b <sub>1</sub>	b <sub>2</sub>	... b <sub>j</sub> ...	b <sub>m</sub>	$\sum a_i$ $\sum b_j$

## 2.2 Balanced and Unbalanced Transportation Problem

A TP is structured as balanced when the total supply from all sources exactly equals the total demand at all destinations. Mathematically, this condition is expressed as:

$$\sum_{i=1}^m P_i = \sum_{j=1}^n Q_j$$

where  $P_1, P_2, \dots, P_m$  shows the supply capacities of various  $m$  sources, and  $Q_1, Q_2, \dots, Q_n$  shows the demand required at various  $n$  destinations.

In case of a unequal between the total demand and the total supply, transportation problem is considered too as unbalanced. To solve this problem, a dummy source or dummy destination is added which having zero transportation cost to balance supply and demand so that required solution techniques can be initiated in proper steps.

In numerical conditions, let  $c_{ij}$  shows the unit transportation cost of transporting products from initial place  $i$  to end place  $j$ . The focus to analyze the quantity to  $x_{ij}$  be shifted from initial point to destination points under the consideration to minimize the cost & time with satisfying all the customers, warehouses.

The main objective to solve the numerical problem by solver to get the minimize cost in unbalanced condition:

$$\text{Min : } \sum \sum c_{ij} x_{ij} \quad (i = 1, 2, 3 \dots, m) \text{ \& } (j = 1, 2, 3 \dots, n)$$

Supply constraints:

$$\sum x_{ij} = p_i \quad (i = 1, 2, 3 \dots, m)$$

$$\text{and } P_1 + P_2 + \dots + P_m > Q_1 + Q_2 + \dots + Q_n$$

Demand constraints:

$$\sum x_{ij} = p_j \quad (j = 1, 2, 3 \dots, n)$$

$$\text{and } P_1 + P_2 + \dots + P_m < Q_1 + Q_2 + \dots + Q_n$$

## 3. Solver's Solution of TP through Excel

In Microsoft Excel, the Solver is an "add on" features provides to opt the best possible optimal solution by using a defined objective function and some of constraints. It is mostly used in the optimization related problems for finance, engineering, and business administration.

To enable and use the Solver function in Excel, proceed as follows:

Go to the "File" tab and choose "Options".

In the Excel Options dialog box, select "Add-ins", then from the "Manage" drop-down, choose "Excel Add-ins" and click "Go".

In the Add-in window, select the box for "Solver Add-in" and click "OK".

After it has been enabled:

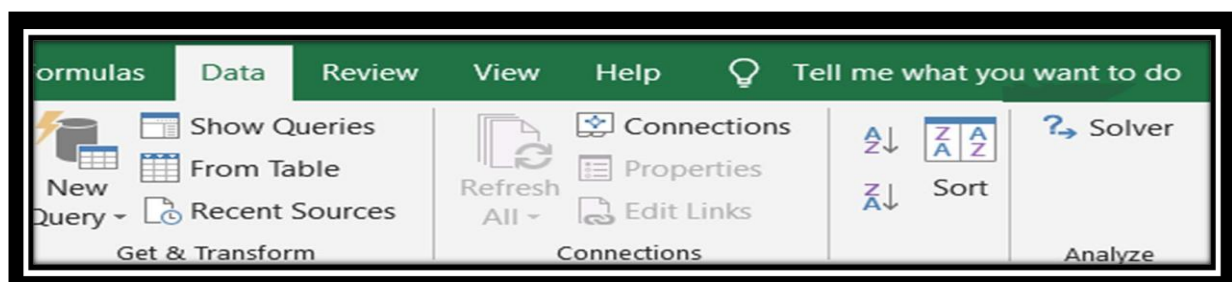
Proceed to the "Data" tab in the ribbon.

Click on the "Solver" choice in the Analysis group.

In Solver Parameters, set the objective, variable cells to adjust, and constraints. Select a appropriate solving method and set parameters such as precision or max iterations where necessary.

Press "Solve" to run the optimization and display the best solution under the given inputs.

Fig. 1: View of Data Tab in Solver



#### 4. Solution Process through Solver & Result

Power Plant's data is given of different warehouses with the different customers related to their transportation:

Per Unit Cost	Cus.-1	Cus.-2	Cus.-3	Cus.-4	Cus.-5	\$
WH-1	2.15	8.20	3.38	8.85	7.42	56000
WH-2	4.39	7.64	8.56	4.00	3.43	40000
WH-3	6.55	2.25	7.20	3.22	7.47	60000
D	13000	34000	75000	35000	11000	

To apply the process for solution by solver, initially fill the data in excel sheet then apply the Solver Method:

C	D	E	F	G	H	I	J	K
	Per unit cost	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5	Available inventory/Supply	
	WH-1	2.15	8.2	3.38	8.85	7.42	56000	
	WH-2	4.39	7.64	8.56	4	3.43	40000	
	WH-3	6.55	2.25	7.2	3.22	7.47	60000	
	Ordered quantity/Demand	13000	34000	75000	35000	11000		

Applying the constraints and conditions to get the optimal solution

C	D	E	F	G	H	I	J	K	L	M
	Per unit cost	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5				
	WH-1	2.15	8.2	3.38	8.85	7.42				
	WH-2	4.39	7.64	8.56	4	3.43				
	WH-3	6.55	2.25	7.2	3.22	7.47				
								Total Cost		0
	Per unit cost	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5	Total	Available inventory		
	WH-1						0	56000		
	WH-2						0	40000		
	WH-3						0	60000		
	Total	0	0	0	0	0	0	156000		
		Customer 1	Customer 2	Customer 3	Customer 4	Customer 5	Total			
	Demand	13000	34000	75000	35000	11000	168000			

**Solver Parameters**

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

## 5. Conclusion

The model related to transportation problem is structured to reduce the expenses involved in transporting raw materials/goods by road, specifically using trucks from their starting place to their intended destinations. In the operational cost breakdown of the mentioned problem of a power plant, transportation plays a vital financial role. A key strategy in tackling transportation problems is focusing on minimizing these logistics costs. This problem can be addressed using the source-to-destination transportation framework.

In this paper, data regarding the movement of raw materials, products from three distinct sources to power plants was manually collected. All correspondent parameters affecting cost were considered in constructing a comprehensive data sheet. This sheet includes financial variables such as usages of diesel, loading coast, toll tax fee, maintenance charges, driver and its assistants' charges.

After settled the data, a transportation model was formulated and analyzed with the objective of optimizing cost efficiency. Various classical transportation solution methods were applied, including the "Row Minimum Method (RMM),

Column Minimum Method (CMM), Vogel's Approximation Method (VAM), North-West Corner Method (NWCN), Least Cost Method (LCM), and Approximation Transportation Method (ATM)". These methods were instrumental in identifying the most cost-effective solution for transporting raw materials.

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